Stand Growth Estimation
Literature Review

Prepared for

Jon Vivian, RPF
Ministry of Forests
Resources Inventory Branch
722 Johnson St.
Victoria BC V8W 9C2

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J.S. Thrower & Associates Ltd. *Consulting Foresters*
Vancouver – Kamloops BC

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Table of Contents

ACKNOWLEGMENTS I

List of Figures II

1. INTRODUCTION 1

2. DEFINITIONS 1

2.1 Tree Classes 1
2.2 Stand Attributes 2
2.3 Growth Types 2

3. GROWTH ESTIMATION SOURCES 3

4. GROWTH ESTIMATORS 4

4.1 Overview 4
4.2 Distance-Independent Probability Selection Non-Compatible Methods 4
4.2.1 Overview 4
4.2.2 Grosenbaugh Estimator 5
4.2.3 Revised Purdue Estimator 5
4.2.4 Furnival Estimator 5
4.3 Distance Independent Probability Selection Compatible Estimators 6
4.3.1 Martin Estimator 6
4.3.2 Van Deusen et al. Estimator 6
4.3.3 Roesch et al. Estimator 6
4.4 Distance Dependent Probability Selection Methods 7
4.4.1 Overview 7
4.4.2 Flewelling Estimator 7
4.4.3 Critical-Height Sampling 7

5. RELATED TOPICS 9

5.1 Relative Efficiency 9
5.2 Sampling with Partial Replacement 9
5.3 Sampling with Moving Strata 10

6. CONCLUSIONS AND RECOMMENDATIONS 10

7. LITERATURE CITED 11

8. APPENDIX I – NUMERICAL EXAMPLE 12

List of Figures

Figure 1. Critical height sampling 8

1. INTRODUCTION

The Ministry of Forests (MOF) Growth and Yield Monitoring Task Force (GYMTF) made recommendations relating to developing growth and yield monitoring protocols which were accepted for implementation by the Forest Productivity Council (FPC). The MOF contracted us to further develop the GYMTF recommendations in the six areas: 1) Volume and Decay; 2) Stand Growth; 3) Treatment Response; 4) Stand Yield; 5) Early Stand Development; and 6) Provincial-level Monitoring (National Forest Inventory (NFI)].

This document is a literature review on the use of fixed-area and variable plots for stand-growth estimation. The report summarizes the review published by Gregoire (1993) and discusses additional topics not covered by Gregoire. This review was prepared by Guillaume Therien, *PhD*.

2. DEFINITIONS

2.1 TREE CLASSES

For this discussion, the following tree classes are recognized in both fixed-area and variable plots:

Survivor: a survivor tree is above minimum size at the beginning of the growth period.

Ingrowth: an ingrowth tree is below minimum size at the beginning of the growth period, and above minimum size at the end of the growth period.

Cut: a cut tree is above minimum size at the beginning of the growth period, but is cut before the end of the growth period. A "C" tree is usually assumed cut immediately after measurement at the beginning of the growth period.

Mortality: a mortality tree is above minimum size at the beginning of the growth period, but is dead before the end of the growth period. An "M" tree is usually assumed dead immediately after measurement at the beginning of the growth period.

For variable plots, two more classes are used to account for trees whose diameter was not big enough to be included in the sample at the beginning of the growth period:

Ongrowth: an ongrowth tree is below minimum size at the beginning of the growth period, above minimum size and at the end of the growth period.

Non-growth: a non-growth tree is above minimum size, but outside the plot at the beginning of the growth period and grows onto the sample plot by the end of the growing period.

The trees present at the second measurement could be summarized as follows:

Beginning of Growth Period	Minimum Size at Beginning of Growth Period	Tree Class
In	Above	Survivor
In	Below	Ingrowth
Out	Above	Nongrowth
Out	Below	Ongrowth

2.2 Stand Attributes

The term "value" will be used to refer to any stand-level attribute. A value is any variable expressed on a per hectare basis (volume, basal area, number of stems, dollars, etc.). A value in fixed-area plots is estimated from:

$$V = \sum_j v_j \times 10{,}000 / a$$

where V is the value/ha, v_j is the value of tree j, and a is the plot area (m^2).

A value in variable plots is estimated from:

$$V = \sum_j BAF \times \frac{v_j}{b_j} = \sum_j BAF \times VBAR_j$$

where BAF is the plot basal area factor, and b_j is the basal area of tree j.

2.3 Growth Types

Growth can be expressed in numerous ways. The definitions used in this paper are based on Husch et al. (1982, p. 291-292):

Gross growth of initial value:	$G_g = (V_2 + M + C - I) - V_1$	V_1 = initial value
Gross growth (including ingrowth):	$G_{g+i} = (V_2 + M + C) - V_1$	V_2 = end value
Net growth of initial value:	$G_n = (V_2 + C - I) - V_1$	M = mortality value
Net growth (including ingrowth):	$G_{n+i} = (V_2 + C) - V_1$	C = cut value
Net increase:	$G_d = V_2 - V_1 = I + S - M - C$	I = ingrowth value
		S = growth of survivor trees

These definitions ignore the growth on M and C trees that occurred after the beginning of the period, and the growth on trees that are smaller than minimum size at the end of the growth period.

3. GROWTH ESTIMATION SOURCES

Tree and stand growth can be estimated from stem-analysis, temporary sample plots, and permanent sample plots (PSPs). Stem-analysis provides tree-level information for survivor trees only. Temporary sample plots provide stand-level yield information and are relatively inexpensive, but have a high coefficient of variation. PSPs provide tree- and stand-growth information and have a lower coefficient of variation than temporary sample plots, but are relatively expensive.

Growth monitoring should focus on stand-level rather than tree-level information. The gain in efficiency of PSPs relative to temporary sample plots should offset the extra cost of PSPs. Consequently, only PSP estimators are reviewed in this document as possible options for the BC growth-monitoring program.

PSPs have existed since at least the middle of the nineteenth century in Europe. Reventlow (1960) mentioned a PSP established in 1812 in a young oak stand. All PSPs were fixed-area until the 1940s. Growth estimators for fixed-area plots were derived using the fixed-area plot-yield estimators and the different expressions of growth (Section 2.3).

Bitterlich (1948, in Bitterlich 1984) introduced the angle-count sampling concept in Europe, and Grosenbaugh (1958) made Bitterlich's ideas known to the English-speaking world, and extended the method to the determination of other attributes than basal area, such as volume. The angle-count sampling method has been known under various names, such as prism plot sampling, variable plot sample, point sampling, plotless sampling, etc. The variable plot uses a different plot size for each individual tree, with plot size proportional to basal area.

A variable plot size means a probability selection, or weight, that varies among trees, proportional to the basal area of the tree. The concept of different probability selections for different trees at a given time was not difficult to accept among foresters. However, the concept of a plot changing size over time (due to basal area growth) was more difficult to accept, which has led many foresters to avoid variable plots in PSPs. Numerous methods have been suggested to convince foresters that variable plots can be used for PSPs. These methods are described in the next section.

4. GROWTH ESTIMATORS

4.1 Overview

Growth estimators can be divided into distance-independent probability selection (DIPS) and distance-dependent probability selection (DDPS), with DIPS estimators divided into non-compatible and compatible methods.

In DIPS non-compatible methods, the yield estimate at the beginning of the growth period plus the growth estimate is not necessarily equal to the yield at the end of the growth period. Compatible methods ensure that the difference between yield estimates equals the growth estimates. The non-compatible methods are generally unbiased but do not try to account for the change in plot size over time. DIPS compatible methods account for the change in plot size over time, but lose part of the information or require more measurements at the beginning of the growth period. DDPS methods account for the change in plot size, do not lose information, and do not require extra measurements.

4.2 Distance-Independent Probability Selection Non-Compatible Methods

4.2.1 Overview

DIPS non-compatible growth methods use the regular yield estimators for M, C, V_1, and V_2:

$$\hat{M} = m$$

$$\hat{C} = c$$

$$\hat{V}_1 = s_1 + m + c$$

$$\hat{V}_2 = i + s_2 + o + n_2$$

where

i = ingrowth estimate from *final* value and *final* basal area
s_1 = survivor estimate from *initial* value and *initial* basal area
s_2 = survivor estimate from *final* value and *final* basal area
m = mortality estimate from *initial* value and *initial* basal area
c = cut estimate from *initial* value and *initial* basal area
o = ongrowth estimate from *final* value and *final* basal area
n_2 = non-growth estimate from *final* value and *final* basal area

The growth of survivor trees is calculated by dividing the difference in value of survivor trees by the basal area at the beginning of the growth periods.

$$S = s_2 - s_1$$

where

s_2 = survivor estimate from *final* value and *initial* basal area

The different DIPS non-compatible methods differ in the way they estimate ingrowth.

4.2.2 Grosenbaugh Estimator

Grosenbaugh (1958) recommended that the growth of nongrowth trees be considered to be zero at first remeasurement and as survivor growth (or cut or mortality) at subsequent remeasurements. His ingrowth estimator used the basal area at the end of the growth period and assumed a minimum diameter at breast height (DBH) threshold of 0 cm.

$$\hat{I}$$

Implementing this method requires identifying non-growth trees by coring the trees at the second measurement or by painting all potential non-growth trees at first measurement. This estimator is biased if the minimum DBH is not 0 cm.

4.2.3 Revised Purdue Estimator

Beers and Miller revised their Purdue estimator (Beers and Miller 1964) in an undated errata sheet, to make it similar to Grosenbaugh (1958). They recognized that the minimum DBH threshold could be larger than 0 cm.

$$\hat{I}$$

The revised Purdue estimator is implemented in the same manner as the Grosenbaugh estimator.

4.2.4 Furnival Estimator

Another estimator, attributed to Furnival[1], excludes all newly qualifying trees, by dividing ingrowth values by the basal area at the beginning of the growth period.

$$\hat{I}$$

where

i' = ingrowth estimate from *final* value and *initial* basal area

This estimator will under-estimate growth if any ingrowth trees are less than 1.3 m in height at first measurement. To implement this estimator, the initial diameter at breast height (DBH) of all potential ingrowth trees must be recorded to calculate their initial tree expansion factor.

[1] Beers and Miller (1964) attributed this estimator to Furnival. However, Furnival said credit should be given to Grosenbaugh (Gregoire 1993).

4.3 Distance Independent Probability Selection Compatible Estimators

4.3.1 Martin Estimator

Martin (1982) recommended using the estimators described in Section 4.1 with a new *I* estimator to ensure additivity among growth components.

$$\hat{I} = \hat{S} - \hat{b} - \hat{h}_2 - \hat{s}_2 + \hat{s}'_2$$

where

s'_2 = survivor estimate from *final* value and *initial* basal area
n_2 = non-growth estimate from *final* value and *final* basal area

A disadvantage of this estimator is that ingrowth estimates cannot be made on individual trees and aggregated, although this is not an issue when individual tree-growth estimates are not required. Also, when ingrowth is very small or non-existent, the ingrowth estimate can be negative due to the change in plot size, resulting from basal area growth.

4.3.2 Van Deusen et al. Estimator

Van Deusen et al. (1986) recommended estimating net increase by using the revised Purdue estimator of ingrowth, but replacing the estimator of survivor growth.

$$\bar{S} = s_2 + s'_1 - h_2$$

They found their estimator to be more precise than the more commonly used $\hat{S}$.

4.3.3 Roesch et al. Estimator

Roesch et al. (1989) discussed a modification to the Van Deusen et al. (1986) estimator by using more information on non-growth trees, and by using a different ingrowth estimator.

$$S^* = s_2 + s'_1 - h_2 + n'_1$$

and

$$I^* = s'_1 + s_1 - h'_1 - \hat{I} - \hat{b}$$

where

s'_1 = survivor estimate from *initial* value and *final* basal area
n_1 = non-growth estimate from *initial* value and *initial* basal area
n'_1 = non-growth estimate from *initial* value and *final* basal area

Roesch et al. showed their estimator to be more efficient in most conditions than the Van Deusen et al. estimator (1986). These two estimators require recording initial diameter and values of potential non-growth trees at the beginning of the growth period.

4.4 Distance Dependent Probability Selection Methods

4.4.1 Overview

In variable plot sampling, BAF is a function of the angle of the prism and the diameter of the tree. The distance between the tree and the sample point does not affect the weight assigned to the value of the tree when calculating the per-hectare value estimate. Distance-dependent methods make use of the distance tree-sample point to give different weights to trees. A tree closer to the sample point will get a higher weight than another more distant tree, even if both trees have the same diameter. The DDPS methods are unbiased and compatible.

4.4.2 Flewelling Estimator

Flewelling (1981) recommended using two prism angles, i.e., two plot sizes, one much larger than the other. This is equivalent to using two plot sizes around a tree. The larger angle subtends a smaller plot (the inner plot); the smaller angle subtends a larger plot (the outer plot). The BAF is constant within the inner plot, but varies with distance if the tree is outside the inner plot, but inside the outer plot. Flewelling's BAF of tree i can be expressed as:

$$BAF_i = \frac{2{,}500 z_i^2}{\ln\left(K / K_L\right) - 1}$$

where

$$z_i = MIN\left(\frac{DBH_i}{d_i}, \frac{2}{K_L}\right)$$

DBH_i = diameter at breast height of tree i
d_i = distance from the tree to the sample point
K_L = cosec of half the large angle (defining the inner plot)
K = cosec of half the small angle (defining the larger plot)

This method is more efficient than DIPS compatible methods. However, it does not eliminate the problem of sudden change due to trees growing onto the plot at the second measurement through increased diameter.

4.4.3 Critical-Height Sampling

Critical-height sampling was developed initially by Kitamura (1962) and developed independently by Iles (1974). The method changes the value-per-unit of basal area ($VBAR_i$) into a ratio that varies with distance ($VBAR(d_i)$), where the average value is the value of the tree, divided by its basal area. Kitamura called this variable value-per-unit of basal area "critical height."

The simplest way to look at critical-height sampling is to imagine a cone centered on a tree whose height is three times the value-per-unit of basal area:

$$H = 3 \times VBAR_j$$

and whose radius R is the radius of the variable plot around the tree (Figure 1). While traditional variable plot sampling flattens the tree value into a disc of radius R, critical-height sampling puts the same value into a cone of radius R. Any other solid with the same volume, such as an expanded tree taper for instance, could be used.

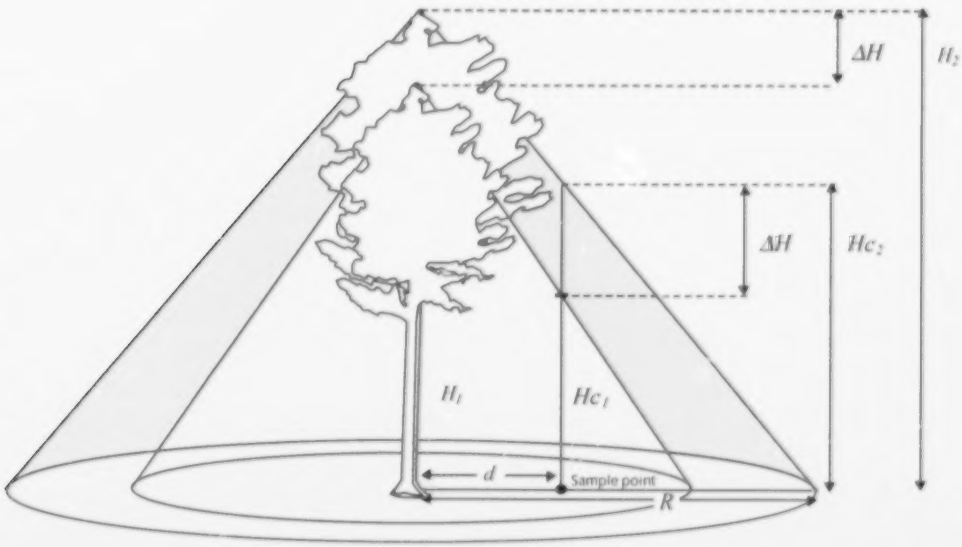

Figure 1. Critical height sampling.

The critical height (H_c) is thus:

$$H_c = 3 \times \frac{1}{\pi R^2} \times \pi d_i^2$$

where R is the radius of the variable plot and d_i is the distance between the tree and the sample point. The average of all possible critical heights is the value-per-unit of basal area ($VBAR_j$). The value/ha is the sum of critical heights times the BAF:

$$V = BAF \sum_{i} H_{ci}$$

Growth is the difference between the critical heights at the beginning and end of the growth period:

$$V_2 - V_1 = BAF_2 \sum_{i} H_{ci2} - BAF_1 \sum_{k} H_{ck2}$$

$$= BAF \sum_{i} H_{ci} \quad \text{(when } BAF_1 = BAF_2\text{)}$$

where BAF_1 and BAF_2 represent the beginning and the end of the growth period respectively. This method does not require keeping track of the different tree classes, since trees not present at the beginning of the growth period had a critical height of 0 m.

A major advantage of this method is that the change from new trees is small and gradual. There is no sudden change from the inclusion of new trees as occurs with traditional variable plot sampling, because only the edge of the cone centered on the new tree initially overlaps the existing cone. The change in critical height can also be predicted based on past growth, allowing a check of predicted growth versus actual growth.

5. RELATED TOPICS

5.1 Relative Efficiency

Martin (1983) showed that the efficiency of permanent-variable relative to permanent fixed-area plots is dependent on the spatial distribution of trees and the correlation between tree growth and basal area. He demonstrated that variable plots become more efficient growth estimators as the correlation between growth and basal area increase. He also demonstrated that growth is better estimated from variable plots when the spatial distribution of trees is more clustered. His study was however very limited in scope.

5.2 Sampling with Partial Replacement

Sampling with partial replacement (SPR) entails estimating growth from a combination of PSPs and independent temporary sample plots. Although SPR offers advantages (cost-effectiveness, more representative sample over time), the disadvantages (overall complexity) probably offset the cost. An expert review panel approved the initial rejection of the idea of using SPR for the Vegetation

Resources Inventory (VRI).[2] This recommendation for the VRI also is valid for the monitoring program.

5.3 Sampling with Moving Strata

Van Deusen (1992) described a method to estimate stratum means in two time periods when the stratum boundaries are changing. This is a special case of SPR because plots that didn't change stratum are considered remeasured plots, and plots that changed stratum are considered temporary sample plots. Sampling moving populations has the same disadvantages as sampling with replacement. The statistical problem associated with moving strata can be avoided by eliminating stratification altogether, or by choosing a stratification system where a stratum can only expand, or only shrink (e.g., natural vs. post-harvest regenerated stands).

6. CONCLUSIONS AND RECOMMENDATIONS

Critical-height sampling based on a cone or other computed taper for tree shape is, in our opinion, the simplest and most adequate option for monitoring stand growth in BC. Bitterlich (1984) considered this critical-height sampling as a major improvement over his own method for estimating stand growth. Many papers have been published on the topic in the official literature. Other unpublished reports also exist. The concept and mathematics of critical height sampling have been worked out and implementing critical height sampling should present little problem.

[2] Rennolls, K., T.G. Gregoire, J. Labau, D. Moon, and J. Osborn. 1995. British Columbia Vegeration Inventory Report of the Expert Review Panel. Unpublished Report, BC Min. Forests, Resources Inventory Branch, Victoria BC.

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8. APPENDIX I – NUMERICAL EXAMPLE

We adapted an example from Husch et al. (1982) to illustrate how critical-height sampling can be used to calculate stand growth (V is an arbitrary value assigned to represent stand growth; see section 2.2). We assumed that a prism with BAF 6 was used at the beginning of the growth period and a BAF-8 prism was used at the end of the growth period (Table 1). Tree 3 shows a negative growth because its critical height decreased with the change in BAF.

Table 1. Critical height sampling example.

Tree No	Distance	Beginning of Growth Period (BAF=6)				End of Growth Period (BAF=8)				Growth ($\Delta V/ha$)
		DBH	*V*	H_c	*V/ha*	*DBH*	*V*	H_c	*V/ha*	
1	2.23	13.9	1.42	60.1	360.5			0	0	-360.5
2	6.52	61.1	81.64	398.6	2391.8	63.8	94.5	442.8	2656.9	265.1
3	0.79	27.1	13.67	609.4	3656.7	28.3	14.21	585.0	3510.2	-146.5
4	2.24	19.4	4.69	206.7	1240.5	21	5.49	227.0	1362.2	121.7
5	5.45	44.1	37.11	287.6	1725.5	44.7	37.94	292.1	1752.4	26.9
6	1.81	14.4	1.53	108.3	649.7	16.6	2.58	166.6	999.6	349.8
7	0.98	10.0	0	0.0	0	12.0	0.8	127.3	763.8	763.8
8	3.02			0	0	16.7	2.49	38.9	233.4	233.4
9	11.11			0	0	55.2	40.59	7.1	42.7	42.7
10	2.28			0	0	12.1	0.75	15.0	90.3	90.3
					10,024.8				*11,411.6*	*1,386.8*